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A Note on Minimizing Batch Flow-time with Capacity Constraints

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ABSTRACT

In this paper, we determine the optimal transfer batch sizes to minimize total batch flow-time when capacity constraints exist on the transfer batch. A four step closed form procedure is developed and proved optimal. Unlike the results of current research, which show that batch sizes decrease in batch order, we prove that, if capacity constraints exist, the batch sizes are only non-increasing in batch order. This result supports the approach taken by heuristic methods employed in lot splitting models which assumes equal batch sizes.

Keywords: optimal transfer gatch sizes, minimum total flow time.

I. INTRODUCTION

The determination of the transfer batch size has received increasing attention with the growing practical concerns of industries to minimize lead times (Smunt et al., 1996). Transfer batches can be obtained by partitioning large orders into smaller batches to move all items more quickly through the production system and then the work-in-process inventory levels can be reduced. The fact is accordance with the just-in-time (JIT) philosophy of making small batches and enhances the interest in its application over the last few years (Chen and Steiner, 1997a, 1997b). Literature in the area can be classified into two broad categories: 1) the determination of optimal transfer batch sizes, and 2) the development of heuristics for determining the transfer batch size. Lee and Chung (1998), Dobson, Karmarkar and Rummel (DKR) (1989, 1987), Naddef and Santos (1984), and Santos and Magazine (1985) have determined optimal transfer batch sizes under varying conditions. The unique aspect of each of these optimal models is that the batch size decreases in batch order in the closed job shop environment.

Furthermore, lot splitting is the process of using transfer batches to move completed portions of a production batch to downstream machines for minimizing the makespan of the schedule and for lowering the work-in-process inventory levels (or, the mean flow time) (Kropp and Smunt 1990; and Baker and Pyke 1990). Szendrovits (1978) formulates a multi-stage, single job problem having equal transfer batches of given size. Under various problem settings, Graves and Kostreva (1987), Vickson and Alfredsson (1992) and Steiner and Truscott (1993) find the optimal lot splitting schedules. Using simulation experiments, Jacobs and Bragg (1988) and Smunt et al. (1996) obtain some results about the performance of the lot splitting decisions respectively. Kropp and Smunt (1990) and Baker and Pyke (1990) developed heuristics utilizing the simplifying assumption that the transfer batch sizes are equal. However, all the studies of the above lot splitting problems assume the same condition of equal batch sizes in their complex models. In this paper, we prove that the optimal transfer batch sizes can be equal when there is capacity constraints on the transfer batch in a closed job shop. In addition, a four step closed form procedure is developed to determine the optimal number of batches and their optimal size. The procedure is proved to be optimal and it can be incorporated into existing heuristic methods to improve performance of the production system.

The problem formulation and the conditions for optimality are discussed in Section 2 of the paper. In Section 3, a closed form solution procedure is developed and proved to be optimal. An implementation example is given in Section 4. Section 5 contains possible extensions of the work including incorporating the procedure into existing heuristics.

II. PROBLEM FORMULATION AND CONDITIONS OF OPTIMALITY

To formulate the batch-flow problem with capacity constraints, we follow DKR in assuming that the following are known and fixed:

d = the number of units to be processed,

s = the setup time for each batch,

r = the processing rate of the machine,

K = capacity limit of the transfer batch,

Likewise, the following decision variables are defined:

n = the number of batches to be run on the machine,

q_i = the quantity produced in the ith batch, i = 1,2, ..., n .

The generalized batch-flow problem (P) is given by:

(P) Minimize $$\sum_{i=1}^{n}\sum_{k=1}^{i}(sq_i + q_k q_i / r) \quad \textbf{(1)}$$

Subject to $\sum_{i=1}^{n} q_i = d$, **(2)**

$q_i \leq K, \ i = 1,2,\ldots,n$, **(3)**

$q_i \geq 0, \ i = 1,2,\ldots,n$. **(4)**

By eliminating constraint (3), problem (P) reduces to the (*BF1*) problem discussed by DKR (1987). Like problem *BF1*, problem (P) is a convex programming problem and the properties of the optimal solution can be derived by using the Karush-Kuhn-Tucker conditions. In the following lemmas $n^* \equiv l$ represents the optimal number of batches and q_i^* represents the optimal size of the i^{th} batch.

Lemma 1. q_i^* is nonincreasing in i.

Proof. See Appendix.

Lemma 2. Problem (P) is equivalent to problem (P1) where P1 is given by:

(P1) Minimize $\sum_{i=1}^{n} \sum_{k=1}^{i} (sq_i + q_k q_i / r)$ **(5)**

Subject to $\sum_{i=1}^{n} q_i = d$, **(6)**

$q_i \leq K, \ i = 1,2,\ldots,n$, **(7)**

$q_i > 0, \ i = 1,2,\ldots,n$. **(8)**

Proof. See Appendix.

Lemma 3, In the optimal solution of problem (P1), if q_i^* is not the last batch size and $q_i^* < K$, then $q_i^* = q_{i+1}^* + sr$.

Proof. See Appendix.

Lemma 4. For problem (P1), if q_l^* is the quantity of the last batch in the optimal solution, then $0 < q_l^* \leq \min\{K, sr\}$.

Proof. See Appendix.

Discussion:

According to Lemma 1, the optimal batch size is nonincreasing in batch order. This result is a refinement of current approaches which conclude that the optimal batch size is decreasing in batch order. If the first batch obtained by DKR's closed form solution is greater than the upper bound of each batch, then there are *l* batches in the optimal solution such that $q_j^* = K$, $j = 1,..., h,$ and $q,$ *is* strictly decreasing in i, where $i = h+1, ..., I$.

Lemma 5. If $q_h^* = K$ and $q_{h+1}^* < K$, then $q_h^* - q_{h+1}^* \le sr$ and $q_i^* = K$ for i = 1,2,...,h.

Proof. See Appendix.

Corollary 1. If $K = \infty$, problem (P) reduces to problem *BF1* .

Proof. See Appendix.

III. THE SOLUTION PROCEDURE

In this section, a solution procedure is developed that satisfies the conditions of optimality expressed in Lemmas 1 through 5. Later, we will prove the procedure to be optimal.

Step 1. If $K \le sr$, then the optimal number of batches is given by $n^* = \left\lceil \frac{d}{K} \right\rceil$,

and theoptimal size of each batch is found by:

$$q_i^* = \begin{cases} K & \text{for } i = 1, 2, ..., n^* - 1, \\ d - (n^* - 1)K & \text{for } i = n^*. \end{cases}$$

If K > sr, go to Step 2.

Step 2. Let $n_1 = \left\lceil \sqrt{\frac{1}{4} + \frac{2d}{sr}} - \frac{1}{2} \right\rceil$. If $K \ge \frac{d}{n_1} + \frac{sr(n_1 - 1)}{2}$ then the optimal number of batches is given by $n^* = n_1$, and the optimal size of each batch is given by:

$$q_i^* = d / n^* + sr(n^* + 1)/2 - i(sr), \qquad \text{for } i = 1, ..., n^* .$$

If $K < \dfrac{d}{n_1} + \dfrac{sr(n_1 - 1)}{2}$, go to Step 3.

Step 3. n Let $n_1 = \left\lfloor \dfrac{K}{sr} \right\rfloor, d' = \dfrac{n_1(n_1+1)sr}{2}$, and $n_2 = \left\lfloor \dfrac{d-d'}{K} \right\rfloor$. If $d - d' - n_2 K = 0$, then the optimal number of batches is given by $n^* = n_1 + n_2$, and the optimal size of each batch is given by:

$$q_i^* = \begin{cases} K & \text{for } i = 1,2,\ldots,n_2, \\ (n^* - i + 1)sr & \text{for } i = n_2+1,\ldots,n^*. \end{cases}$$

If $d - d' - n_2 K > 0$, go to Step 4.

Step 4. Let $e = K - n_1 sr$ and $u = \dfrac{d - d' - n_2 K}{n_1 + 1}$. If $u \le e$, then the optimal number of batches is given by $n^* = n_1 + n_2 + 1$, and the optimal batch size is given by:

$$q_i^* = \begin{cases} K & \text{for } i = 1,2,\ldots,n_2, \\ (n^* - i)sr + u & \text{for } i = n_2+1,\ldots,n^*. \end{cases}$$

If $u > e$, then the optimal number of batches is given by $n^* = n_1 + n_2 + 1$, *an*d the optimal batch size is given by:

$$q_i^* = \begin{cases} K & \text{for } i = 1,2,\ldots,n_2, \\ (n^* - i)sr + q_{n^*}^* & \text{for } i = n_2+1,\ldots,n^*, \end{cases}$$

where $q_{n^*}^* = \dfrac{d}{n_1} - \dfrac{n_1 - 1}{2} sr - \dfrac{n_2 + 1}{n_1} K$.

Theorem 1. The above 4 step procedure provides an optimal solution for problem P.

Proof. See Appendix.

IV. AN EXAMPLE

This example includes five problems. For each problem, d = 150, s = 5, and r = 3, where d, s, and r denote the values for demand, setup time, and processing rate. Problems 1 through 5 have capacity constraints on the transfer batch of 12, 60, 35, 42, 32 respectively.

Problem 1: K= 12

Step 1. Since $sr = 15 \geq K = 12,$ the optimal solution is

$$n^* = \left\lceil \frac{d}{K_1} \right\rceil = \left\lceil \frac{150}{12} \right\rceil = 13,$$

$$q_i^* = \begin{cases} 12 & \text{for } i = 1, 2, \ldots, 12, \\ 6 & \text{for } i = 13. \end{cases}$$

Problem 2: K = 60

Step 1. Since sr = 15 < K = 60, go to Step 2.

Step2. Let $n_1 = \left\lceil \sqrt{\frac{1}{4} + \frac{2d}{sr}} - \frac{1}{2} \right\rceil = \left\lceil \sqrt{\frac{1}{4} + \frac{2*150}{5*3}} - \frac{1}{2} \right\rceil = 4.$ Since $K \geq 150/4 + 15*3/2 = 60,$ the optimal number of batches is $n^* = n_1 = 4.$ The optimal batch size is

$$q_i^* = 150/4 + 15*5/2 - i(15) = 75 - 15i, \qquad \text{for } i = 1, \ldots, 4.$$

Problem 3: K = 35

Step 1. Since sr = 15 < K= 35, go to Step 2.

Step 2. Let $n_1 = \left\lceil \sqrt{\frac{1}{4} + \frac{2d}{sr}} - \frac{1}{2} \right\rceil = 4.$ Since K = 35 < 60, go to Step 3.

Step 3. Let $n_1 = \left\lfloor \frac{K}{sr} \right\rfloor = 2$, $d' = \frac{n_1(n_1+1)sr}{2} = \frac{2*3*15}{2} = 45$,

and $n_2 = \left\lfloor \frac{d-d'}{K} \right\rfloor = \left\lfloor \frac{150-45}{35} \right\rfloor = 3.$ Since $d - d' - n_2 K = 150 - 45 - 3*35 = 0$, the optimal number of batches is

$n^* = 2 + 3 = 5$, and the optimal batch size is

$$q_i^* = \begin{cases} 35 & \text{for } i = 1, 2, 3, \\ (6 - i)*15 & \text{for } i = 4, 5. \end{cases}$$

Problem 4: K = 42

Step 1. Since sr = 15 < K = 42, go to Step 2.

Step 2. Let $n_1 = \left\lceil \sqrt{\frac{1}{4} + \frac{2d}{sr}} - \frac{1}{2} \right\rceil = 4.$. Since K = 42 < 60, go to Step 3.

Step 3. Let $n_1 = \left\lfloor \frac{K}{sr} \right\rfloor = 2$, $d' = \frac{n_1(n_1+1)sr}{2} = \frac{2*3*15}{2} = 45$,

and

$n_2 = \left\lfloor \frac{d-d'}{K} \right\rfloor = \left\lfloor \frac{150-45}{42} \right\rfloor = 2.$ Since $d - d' - n_2 K = 150 - 45 - 2*42 = 21 > 0$, go to Step 4.

Step 4. Let $e = K - n_1 s r$ = 42 – 2 * 15 =12 and $u = \frac{d - d' - n_2 K}{n_1 + 1} = \frac{150 - 45 - 2*42}{2+1} = 7$.

Since $u = 7 \leq e = 12$, the optimal number of batches is $n^* = n_1 + n_2 + 1 = 5$, and the optimal batch size is

$$q_i^* = \begin{cases} 42 & \text{for } i = 1, 2, \\ (5 - i)*15 + 7 & \text{for } i = 3, 4, 5. \end{cases}$$

Problem 5: K = 32

Step 1. Since $sr = 15 < R = 32$, go to Step 2.

Step 2. Le t $n_1 = \left\lceil \sqrt{\frac{1}{4} + \frac{2d}{sr}} - \frac{1}{2} \right\rceil = 4$. Since $K = 32 < 60$, go to Step3.

Step 3. Let $n_1 = \left\lfloor \frac{K}{sr} \right\rfloor = 2$, $d' = \frac{n_1(n_1 + 1)sr}{2} = \frac{2*3*15}{2} = 45$, and

$n_2 = \left\lfloor \frac{d - d'}{K} \right\rfloor = \left\lfloor \frac{150-45}{32} \right\rfloor = 3$. Since

$d - d' - n_2 K = 150 - 45 - 3*32 = 9 > 0$, go to Step 4.

Step 4. Let $e = K - n_1 s r = 32 - 2*15 = 2$ and $u = \frac{d - d' - n_2 K}{n_1 + 1} = \frac{150-45-3*32}{2+1} = 3$.

Since $u = 3 > e = 2$, the optimal number of batches is $n^* = n_1 + n_2 + 1 = 6$, and the optimal batch size is

$$q_i^* = \begin{cases} 32 & \text{for } i = 1, 2, 3, 4, \\ (6 - i)*15 + 3.5 & \text{for } i = 5, 6 \end{cases}$$

where $q_{n^*}^* = \frac{d}{n_1} - \frac{n_1 - 1}{2} sr - \frac{n_2 + 1}{n_1} K = \frac{150}{2} - \frac{2-1}{2} * 15 - \frac{3+1}{2} * 32 = 3.5.$

V. SUMMARY AND CONCLUSIONS

In this paper, we derived the conditions for the determination of the optimal transfer batch size to minimize total flow-time when there is capacity constraints on the transfer batch. The closed form procedure developed is easily applied. Furthermore, the procedure can be incorporated into current heuristics of the batching problems or the lot splitting problems. The four step procedure can replace steps 2, 3a, 3b and 4 of algorithm A reported by Baker and Pyke [1, pp. 482] . This heuristic algorithm first finds that machine with the largest processing time. This is equivalent to the processing time used in our closed form procedure. For an *m* machine problem, if a better lower bound with respect to the objective function is constructed, then the procedure discussed in Section 3 can be applied to each heuristic of the previous research results (Lee and Chung, 1998; and DKR 1989, 1987). Although the lower bound of each previous research is still useful for each situation, the analysis of each optimal solution shows that it is possible to develop a better lower bound for each case as capacity constraints exist on the transfer batch. The performance of the modified heuristics will be compared with the better lower bound and the efficiency of this approach will be investigated in future research.

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APPENDIX

Proof of Lemma 1. The proof of Lemma 1 is by contradiction. Suppose there are two batches q_i^* and q_{i+1}^* in the optimal solution such that $q_{i+1}^* > q_i^*$. The optimal solution can be improved by exchanging the order of these two batches to build another batch sequence. Without loss of generality, assume that the i^{th} batch starts at time t_{i-1}. By letting *FT1* and *FT2* denote the batch-flow time of the first batching sequence and the second batching sequence respectively, the difference between the above flow times only occurs at the i^{th} batch and the $(i+1)^{th}$ batch.

The following expression can be obtained:

$$\text{FT1 - FT2} = \left(\frac{q_i^*}{r}+s+t_{i-1}\right)q_i^* + \left(\frac{q_{i+1}^*}{r}+s+\frac{q_i^*}{r}+s+t_{i-1}\right)q_{i+1}^*$$

$$-\left[\left(\frac{q_{i+1}^*}{r}+s+t_{i-1}\right)q_{i+1}^* + \left(\frac{q_{i+1}^*}{r}+s+\frac{q_i^*}{r}+s+t_{i-1}\right)q_i^*\right]$$

$$= s\,q_{i+1}^* - s\,q_i^*$$

$$= s\,(q_{i+1}^* - q_i^*).$$

Since $q_{i+1}^* > q_i^*$, the above difference is positive. This implies that *FT1* is greater than *FT2* which contradicts the assumption of *FT1* being optimal. Consequently, $q_i^* \geq q_{i+1}^*$.

Proof of Lemma 2. From Lemma 1, the optimal batch size is nonincreasing in batch order. Note that if the batch size is zero, the flow time is also zero. By eliminating the condition, $q_i = 0$, the optimal solution of problem (P) can be obtained by solving problem (P1).

Proof of Lemma 3. Since problem (P) is a convex programming problem, it can be restated as:

(P') Minimize $\sum_{i=1}^{n} \sum_{k=1}^{i} (sq_i + q_k q_i / r)$ **(1)**

Subject to $\sum_{i=1}^{n} q_i - d = 0$, $:\lambda$ **(2)**

$q_i - K \le 0$, $:\mu_i, i = 1, 2, \ldots, n$, **(3)**

$q_i \ge 0$, $:\gamma_i, i = 1, 2, \ldots, n$, **(4)**

where λ, μ_i and γ_i are Lagrangian multipliers. The first order conditions are:

$$s\,i + \frac{q_i}{r} + \frac{d}{r} - \lambda + \mu_i - \gamma_i = 0,$$

$$(q_i - K)\mu_i = 0,$$

$$q_i \gamma_i = 0,$$

$$\lambda \ge 0, \mu_i \ge 0, \text{ and } \gamma_i \ge 0, \; i = 1, 2, \ldots, n.$$

From Lemma 1 and $q_i^* < K$, it is obvious that for optimality, $\mu_i = 0$, $\gamma_i = 0$ and

$$q_i^* = \lambda r - d - i(sr). \quad \text{Hence } q_i^* = q_{i+1}^* + sr.$$

Proof of Lemma 4. Two relationships exist between K and sr. If $K \le sr$, then $q_l^* \le K$ is necessary to satisfy constraint (7). If K > sr, the following relations are obtained from

Lemma 3: $q_l^* = \lambda r - d - \mu_l r - l(sr)$,

$$q_{l+1}^{*} = 0 = \lambda r - d - \gamma_{l+1} r - (l+1)(sr),$$

which yields $q_{l}^{*} = sr - \mu_{l} r - \gamma_{l+1} r.$

Because of the nonnegativity restrictions on μ_l and γ_{l+1}, $r > 0$ and $q_l^{*} \le sr$ results. Hence, $0 < q_l^{*} \le \min\{K, sr\}$.

Proof of Lemma 5. Lemma 5 is proved by contradiction. Assume $q_h^{*} - q_{h+1}^{*} > sr$ results in a flow time ($FT1$) . Then there exists a $\delta > 1$ such that $q_{h+1}^{*} = K - \delta sr$. By setting $\delta' = \frac{\delta - 1}{2}$, we can move $\delta' sr$ pieces from the h^{th} batch to the $(h+1)^{th}$ batch to build a second sequence with a flow time ($FT2$) . Without loss of generality, assume the h^{th} batch starts at time t_{h-1}. The difference between $FT1$ and $FT2$ is:

FT1-FT2=

$$[(\frac{q_{h+1}^{*} + sr + 2\delta' sr}{r} + s + t_h)(q_{h+1}^{*} + sr + 2\delta' sr) + (\frac{2q_{h+1}^{*} + sr + 2\delta' sr}{r} + 2s + t_h)q_{h+1}^{*}]$$

$$-[(\frac{q_{h+1}^{*} + sr + \delta' sr}{r} + s + t_h)(q_{h+1}^{*} + sr + \delta' sr) + (\frac{2q_{h+1}^{*} + sr + 2\delta' sr}{r} + 2s + t_h)(q_{h+1}^{*} + \delta' sr)]$$

$$= (\delta')^2 s^2 r.$$

Because $(\delta')^2$, s^2 and r are all positive, the difference, $FT1$-$FT2$, is greater than zero which contradicts the assumption. Hence, $q_h^{*} - q_{h+1}^{*} \le sr$.

Proof of Corollary 1. The proof is quickly shown by noting that if $K = \infty$, the transfer batch does not have capacity constraints. Consequently, constraint 3 is eliminated in problem (P), which is then equivalent to problem $BF1$.

Proof of Theorem 1. Three relationships exist between sr, K and q_1 that determine the optimal number of batchs and the batch sizes. These relationships are:

(1) If $K \le sr$, then only one feasible solution satisfies Lemmas 1 through 4. This solution is found in Step 1.

(2) If $K > sr$ and $q_1 \le K$, then the second relation exists which is equivalent to problem *BF1* of DKR. The solution procedure is the same as that obtained by DKR and is Step 2 of the procedure.

(3) If $K > sr$ and $q_1 > K$, Steps 3 and 4 find the unique optimal solution. By applying the results of Lemmas 1 through 5, three different cases are checked in Steps 3 and 4. After allocating the quantity d' to n_1 batches such that $q_i \le K$, the quantity of the remaining work, $d - d'$, is allocated to the first n_2 batches such that $q_i = K, i = 1, 2, \ldots, n_2$. Define e as the difference between q_{n_2+1} and $q_{n_2} = K$. For the first case, if $d - d' - n_2 K = 0$, then the optimal solution is found in Step 3. If $d - d' - n_2 K > 0$, then the remaining work is reallocated to the last n_1 batches, increasing the number of batches by one. Since u is the average remaining work, if $u \le e$, the optimal solution is found in Step 4. If $u > e$, then $q_{n_2+1} > K$. In this case, e pieces of work are first allocated to the t $(n_2+1)^{th}$ batch such that $q_{n_2+1} = K$. The remaining work then equals $d - d' - n_2 K - e$ and it is allocated to the last $n_1 - 1$ batches. This increases the number of batches by one. After adding $\frac{d - d' - n_2 K - e}{n_1}$ to the last n_1 batches, the optimal solution is obtained in Step 4.

產能限制的批量流程時間極小化之探討

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摘要

針對產能限制 (capacity constraints) 的轉換批量 (transfer batch) 問題，本論文乃在確定最佳化的轉換批量大小以極小化總批量流程時間 (total batch flow-time)。本研究發展出一個具有四步驟封閉型式的程序 (closed form procedure) 並且經過證明其解答是最佳決策。不同於目前的研究結果 (其最佳化的批量大小乃隨批量次序遞減)，本論文證明：當存在產能限制時，最

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佳化的批量大小則僅是隨著批量次序有非遞增的事實。此結果支持了批次分散模式 (lot splitting model) 中所使用的啟發式方法 (此法乃假定最佳化的批量是等批量大小)。

關鍵詞彙：最佳化的轉換批量，極小化總批量流程時間